Exercise 1.3

1 Write without brackets.

(a) $(x+5)^2$ (b) $(3x-2)^2$ (c) $(3x+4)(3x-4)$

2 Simplify the following equations into the form $ax+by+c=0$.

(a) $(x+3)^2+(y+4)^2=(x-2)^2+(y-1)^2$

(b) $(2x+1)^2+(y-3)^2=(2x+3)^2+(y+1)^2$

3 Simplify the following where possible.

(a) $\sqrt{x^2+4}$ (b) $\sqrt{x^2-4x+4}$ (c) $\sqrt{x^2-1}$

(d) $\sqrt{x^2+9x}$ (e) $\sqrt{x^2-y^2}$ (f) $\sqrt{x^2+2xy+y^2}$

4 Write the following in the form $(x+a)^2+b$.

(a) $x^2+8x+19$ (b) $x^2-10x+23$ (c) x^2-5x-6

5 Factorise as fully as possible.

(a) x^2-25 (b) $4x^2-36$ (c) $4x^2-9y^4$

(d) $3x^2-7x+2$ (e) $3x^2-5x+2$ (f) $6x^2-5x-6$

Further Maths Only

6* Multiply out and simplify.

(a) $\left(x+\frac{1}{x}\right)^2$ (b) $\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$ (c) $\left(x+\frac{2}{x}\right)\left(x-\frac{3}{x}\right)$

Exercise 1.3

(1) (a) $(x+5)^2 = x^2 + 5x + 5x + 25$

$= x^2 + 10x + 25$

$\times$	x	$+5$
x	x^2	$+5x$
$+5$	$+5x$	$+25$

(b) $(3x-2)^2 = 9x^2 - 6x - 6x + 4$

$= 9x^2 - 12x + 4$

$\times$	$3x$	-2
$3x$	$9x^2$	$-6x$
-2	$-6x$	$+4$

(c) $(3x+4)(3x-4) = 9x^2 + 12x - 12x - 16$

$= 9x^2 - 16$

$\times$	$3x$	$+4$
$3x$	$9x^2$	$+12x$
-4	$-12x$	-16

Note: This is a difference of two squares so the answer can be written down without the intermediate step

(2) (a) $(x+3)^2 + (y+4)^2 = (x-2)^2 + (y-1)^2$

$x^2 + 6x + 9 + y^2 + 8y + 16 = x^2 - 4x + 4 + y^2 - 2y + 1$

$(-x^2, -y^2)$ $\quad 6x + 8y + 25 = -4x - 2y + 5$

$(+4x, +2y - 5)$ $\quad 10x + 10y + 20 = 0$

$(\div 10)$ $\quad \underline{x + y + 2 = 0}$

(b) $(2x+1)^2 + (y-3)^2 = (2x+3)^2 + (y+1)^2$

$4x^2 + 4x + 1 + y^2 - 6y + 9 = 4x^2 + 12x + 9 + y^2 + 2y + 1$

$(-4x^2, -y^2)$ $\quad 4x - 6y + 10 = 12x + 2y + 10$

$0 = 8x + 8y$

$(-4x, +6y, -10)$

$(\div 8)$ $\quad \underline{x + y = 0}$

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(3) (a) $\sqrt{x^2+4}$ cannot be simplified

(b) $\sqrt{x^2-4x+4} = \sqrt{(x-2)^2} = \underline{x-2}$

(c) $\sqrt{x^2-1}$ cannot be simplified

(d) $\sqrt{x^2+9x}$ cannot be simplified

(e) $\sqrt{x^2-y^2}$ cannot be simplified

(f) $\sqrt{x^2+2xy+y^2} = \sqrt{(x+y)^2} = \underline{x+y}$

4

(a) $x^2 + 8x + 19 = (x+4)^2 - 16 + 19$

$= (x+4)^2 + 3$

(b) $x^2 - 10x + 23 = (x-5)^2 - 25 + 23$

$= (x-5)^2 - 2$

(c) $x^2 - 5x - 6 = \left(x - \frac{5}{2}\right)^2 - \frac{25}{4} - 6$

$= \left(x - \frac{5}{2}\right)^2 - \frac{25}{4} - \frac{24}{4}$

$= \left(x - \frac{5}{2}\right)^2 - \frac{49}{4}$

5

(a) $x^2 - 25 = x^2 - 5^2$ difference of two squares

$= (x+5)(x-5)$

(b) $4x^2 - 36 = 4(x^2 - 9)$

$= 4(x^2 - 3)^2$

$= 4(x+3)(x-3)$

OR $4x^2 - 36 = (2x)^2 - 6^2$

$= (2x+6)(2x-6)$

$= 2(x+3) \cdot 2(x-3)$

$= 4(x+3)(x-3)$

(c) $4x^2 - 9y^4 = (2x)^2 - (3y^2)^2$

$= \underline{(2x + 3y^2)(2x - 3y^2)}$

(d) $3x^2 - 7x + 2 = 3x^2 - 6x - 1x + 2$

$= 3x(x-2) - 1(x-2)$

$= \underline{(x-2)(3x-1)}$

P: $3 \times 2 = +6$, A: -7 } -6 and -1

(e) $3x^2 - 5x + 2 = 3x^2 - 3x - 2x + 2$

$= 3x(x-1) - 2(x-1)$

$= \underline{(x-1)(3x-2)}$

P: $3 \times 2 = +6$, A: -5 } -3 and -2

(f) $6x^2 - 5x - 6 = 6x^2 - 9x + 4x - 6$

$= 3x(2x-3) + 2(2x-3)$

$= \underline{(2x-3)(3x+2)}$

P: $6 \times -6 = -36$, A: -5 } -9 and $+4$

If you are not familiar with the method used above it is often called PAFF

P stands for product (multiply the coefficent of x^2 by the constant term). A stands for addition (this is the coefficent of x). We need to find two integers with product P and sum A. We then split the x term using these two integers. Then F (factorise) the first two terms and the last two terms. Then F (final factors).

FURTHER MATHEMATICS ONLY

6 (a) $\left(x+\frac{1}{x}\right)^2 = x^2 + 2x\cdot\frac{1}{x} + \left(\frac{1}{x}\right)^2$

$$= x^2 + 2 + \frac{1}{x^2}$$

(b) $\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right) = x^2 - \left(\frac{1}{x}\right)^2$ (difference of two squares)

$$= x^2 - \frac{1}{x^2}$$

(c) $\left(x+\frac{2}{x}\right)\left(x-\frac{3}{x}\right) = x^2 + 2 - 3 - \frac{6}{x^2}$

$$= x^2 - 1 - \frac{6}{x^2}$$

$\times$	$x + \frac{2}{x}$
x	$x^2 \quad +2$
$-\frac{3}{x}$	$-3 \quad -\frac{6}{x^2}$